From Chapter 9: "Getting the 3 R's Right"

in The Schools Our Children Deserve (Boston: Houghton Mifflin, 1999)

What Works Better than Traditional Math Instruction

Why the Basics Just Don't Add Up

By Alfie Kohn

The still-dominant Old School model begins with the assumption that kids primarily need to learn "math facts": the ability to say "42" as soon as they hear the stimulus "6 x 7," and a familiarity with step-by-step procedures (sometimes called algorithms) for all kinds of problems — carrying numbers while subtracting, subtracting while dividing, reducing fractions to the lowest common denominator, and so forth.

Once the subject is defined this way, there isn't much mystery as to what technique will be used. "When the process of learning in arithmetic is conceived to be the mere acquisition of isolated, independent facts, the process of teaching becomes that of administering drill."[1] You do one problem after another until you've got it down cold. It may make you dread the whole subject (and avoid it whenever possible), but that's the way it has to be done. Moreover, doing math is pretty much the same in a high-school algebra lesson as it is in a first-grade addition lesson. The teacher begins by demonstrating the right way to do a problem, then assigns umpteen examples of the same problem (except with different numbers), the idea being for students to imitate the method they were shown, with the teacher correcting their efforts as necessary.

If students have trouble producing the right answer, that is "taken as evidence only of the need of further drill."[2] This, as we've seen, is exactly how the failure of direct phonics instruction is explained away: the more it doesn't work, the more you obviously need it. And, also like traditional ways of teaching children to read, most math classrooms are predicated on the transmission model: students are simply given facts and procedures by the teacher and the textbook. Math textbooks, in fact, are often regarded by student and teacher alike as the source of Truth – "a cryptic but authoritative document" — such that everyone's job is to figure out "what it wants you to do."[3]

One math lesson, or teacher, differs from the next only in terms of relatively trivial issues: the number of problems to be done at the blackboard versus at one's seat versus at home, the clarity of the teacher's explanations, or the difficulty of the calculations in each problem. The decisions that really matter have already been made: seeing math as a collection of truths "out there" that have to be instilled in students by way of repetitive drill. These elements are present even in classes that are widely judged to be of high quality, with respected teachers, attentive students, and good results on standardized tests.[4] "Mindless mimicry mathematics," as the National Research Council calls it,[5] has come to be accepted as the norm in our schools. One consequence of this is on display every time an adult casually describes himself or herself as hating math and lacking any aptitude for it. Generations of ex-students have written off the subject — as well as their own competence – at least partly as a result of Old School instruction.

More than 70 years ago, a math educator named William Brownell observed that “intelligence plays no part” in this style of teaching. Even now, most students are still being taught math “as a routine skill,” says Lauren Resnick. “They do not develop higher order capacities for organizing and interpreting information.”[6] Thus, students may memorize the fact that 0.4 = 4/10, or successfully follow a recipe to solve for x, but the traditional approach leaves them clueless about the significance of what they’re doing. Without any feel for the bigger picture, they tend to plug in numbers mechanically as they follow the technique they’ve learned. They couldn’t be described as “successful in quantitative thinking,” because for that, as Brownell explained, “one needs a fund of meanings, not a myriad of ‘automatic responses’. . . . Drill does not develop meanings. Repetition does not lead to understandings.”[7]

As a result of the standard approach to math instruction, students often can’t take the methods they’ve been taught and transfer them to problems even slightly different from those they’re used to. For example, a seven-year-old may be a whiz at adding numbers when they’re arranged vertically on the page, but then throw up her hands when the same problem is written horizontally. Or she may possess a “rich informal knowledge base derived from working with quantities in everyday situations”[8] that allows her to figure out how many cookies she would have if she started out with 16 and then received nine more – but regard that understanding as completely separate from the way you’re supposed to add in school (where she may well get the wrong answer).[9]

Math educators are constantly finding examples of how kids can do calculations without really knowing what they’re doing. Children given the problem

$$\frac{274+274+274}{3}$$

set about laboriously adding and then dividing, missing the fact that they needn’t have bothered — a fact that would be clear if they really understood what multiplication and division are all about.[10] One researcher gave problems like 7+52+186 to second, third, and fourth graders, and found that those who had been taught in the usual way — with a set procedure for carrying numbers from the ones place to the tens place, and then from the tens place to the hundreds place – didn’t just make mistakes: they made mistakes so outlandish as to suggest a complete failure to grasp the quantities involved. The answers from these students included 29, 30, 989, and 9308.[11] (As we’ll see later, students of the same age who were taught nontraditionally — without textbooks, worksheets, or any presentation by the teacher of the “right way” of adding — did far better.)

Another striking example comes from the way 13-year-olds dealt with a problem that appeared in the National Assessment of Educational Progress (NAEP). The question was: “An army bus holds 36 soldiers. If 1,128 soldiers are being bused to their training site, how many buses are needed?” If you divide the first number into the second, you get 31 with a remainder of 12, meaning that 32 buses would be required to transport all the soldiers. Most students did the division correctly, but fewer than one out of four got the question right. The most common answer was “31 remainder 12.”[12]

This sort of robotic calculation doesn’t reflect a mental defect in the students but the triumph of the back-to-basics, drill-and-skill model of teaching math. And that’s not just one person’s opinion. Analysts of NAEP data for the Educational Testing Service observed that students can “recite rules” but often don’t “have any idea whether their answers are reasonable.” The “preposterous answers” that often result can be attributed to “a general overemphasis in contemporary curricula on computation-related skills or the tendency to teach skills and knowledge before integrating applications and problem solving into instruction."[13]

In some circumstances, it may be useful for students to practice a skill – after they've come to thoroughly understand the underlying idea. But when drill starts too early or takes up too much class time, it offers precious little benefit to the students who are already successful,[14] and does absolutely nothing for those who aren't, except to make them feel even more incompetent. Indeed, it's most important to avoid this scripted approach in the case of students who have trouble making sense of what's going on: the more they're given algorithms and told exactly what to do, the farther behind they fall in terms of understanding.[15] By now we should hardly be surprised to find that U.S. schools get things precisely backwards, subjecting these students in particular to an endless regimen of drills.

Math Worth Doing

All of this has been noticed by people who make their living thinking about how math ought to be taught. Several documents for reforming the field, including, most notably, the standards disseminated in 1989 by the National Council of Teachers of Mathematics (NCTM), have said that math classes should revolve around making meaning (just as with nontraditional approaches to reading and writing) and promote thinking rather than rule-memorizing. Students should be encouraged to write and talk about their ideas, to understand the underlying concepts and be able to put them into words. They should spend their time

> abstracting, applying, convincing, classifying, inferring, organizing, representing, inventing, generalizing, specializing, comparing, explaining, patterning, validating, proving, conjecturing, analyzing, counting, measuring, synthesizing, and ordering [because] these are the sorts of activities that are thought to characterize the work of mathematicians.[16]

Students in classrooms where mathematical thinking is encouraged from a very young age learn how to estimate and predict. ("How many pencils do you think there are in the whole school? Is there a way we could know for sure without counting?") They acquire basic skills in the process of solving meaningful problems — often with their peers. They may use calculators, as adults often do, so that they can tackle more challenging and engaging problems than would be possible if they had to direct their energy to computation. In contrast to a classroom whose main activities are listening to the teacher and filling out worksheets, such a learning environment is distinguished by students "sitting in groups, discussing ideas, doing experiments, making diagrams, using concrete objects to test their conjectures, following blind alleys, and now and then experiencing the satisfaction of discovering something they did not know before."[17]

Several features of such classrooms should sound quite familiar by now. For one thing, cooperation is an important part of learning. Sometimes that takes the form of children solving problems in pairs, followed by a whole-class discussion. Teachers make a special effort to create a caring community lest disagreeing with each other's answers turn into a competition and discourage some children from speaking out. For another thing, students are more active, more at the center of the classroom, than in the traditional model; their choices count and their voices are heard. And as with Whole Language, which takes advantage of naturally occurring uses of words and ideas, math teachers are always on the look-out for real issues and activities that can offer "opportunities for children to engage in numerical reasoning."[18] News articles from the morning paper raise questions about probability; cooking provides authentic fraction problems; even taking attendance can be put to use ("What proportion of our class is absent?").

Some of us will immediately find all of this appealing, perhaps because we see it a refreshing contrast to the pointless tedium we had to endure in math class way back when. But others will dismiss this vision for exactly the same reason: it's not what they're used to. Even parents who are open to being convinced of the value of Whole Language may seem skeptical about conceptual approaches to math. The idea of reading for understanding is clear enough (few adults, after all, spend their time underlining topic sentences or circling vowels), but how many of us have any experience with math instruction that emphasizes understanding?[19] We think of math as a subject where you churn out answers that are either right or wrong, and we may fear that anything other than the conventional drill 'n skill methods will leave our kids unable to produce the correct answers when it comes time for them to take a standardized test. Indeed, it asks a lot for people to support, or even permit, a move from something they know to something quite unfamiliar.

Nevertheless, that's precisely what most experts in the field are asking us to do – and with good reason. As with any of the issues discussed in this book, there are basically three ways to convince skeptics. First, there's theory: explaining the objectives of, and the rationale for, doing things differently. That's what I've been trying to do in this section and the preceding one. Second, there's research, which will be reviewed in Appendix A [reprinted below]. Finally, there are examples, ideally gleaned from first-hand observation of extraordinary classrooms – or, next best, descriptions that give a flavor of what these ideas look like in practice and how they compare to the usual fare.

Consider, then, a teacher who tells her students what a "ratio" is, expecting them to remember the definition. Now imagine a teacher who has first graders figure out how many plastic links placed on one side of a balance are equivalent to one metal washer on the other side. Then, after discovering that the same number of links must be added again to balance an additional washer, the children come to make sense of the concept of ratio for themselves. Which approach do you suppose will lead to a deeper understanding?

Consider a classroom where third graders open their math textbooks to the contrived "word problems" on page 39. ("A train leaves Washington, D.C. heading west at 65 m.p.h....") Now imagine a classroom where students are asked to compare the weight of two pieces of bubblegum (with and without sugar) before and after each piece has been chewed — making predictions, recording results, explaining the differences, all the while adding, subtracting, multiplying, dividing, as well as using decimals and percentages, learning to estimate and extrapolate. In which classroom are they more likely to see math as relevant, appealing, and something at which they can be successful?

Consider a ditto full of fraction problems for upper-elementary level students: "1/2 + 1/3 = ___" and so on. Now imagine that they are instead asked to explain, using words and numbers, "Why doesn't 1/2 + 1/3 equal 1/5?" Which question do you suppose will give the teacher a better sense of how each student is thinking?

Finally, consider – which is to say, remember — a conventional middle-school or high school math curriculum. Now imagine a class where the students' year-long assignment, while learning various concepts, is to collaborate on writing a textbook for the students who will take their place in the same room next year. Which is more rigorous? Which do you want for your child?[20]

Inventing Facts

It's a pretty sharp contrast, between math defined principally in terms of skills and math defined principally in terms of understanding. But if we are persuaded by a constructivist account of learning, even the latter isn't enough. When traditionalists insist that it's most important for kids to "know their math facts," we might respond not only by challenging those priorities but by asking what is meant by know. The key question is whether understanding is passively absorbed or actively constructed. In the latter case, math actually becomes a creative activity.

In early 1998, an op-ed article in the New York Times mentioned the value of having students design and carry out their own experiments in mathematics. "Just think," a woman sarcastically replied in a letter to the editor, by doing that, "students might reinvent the Pythagorean theorem." She concluded: maybe this way we can "shoot for last place next year" in international comparisons.[21] Never mind that the poor showing for U.S. students in the most recent tests was likely a result of the very "facts and skills" curriculum that this letter-writer preferred. (She can be excused for ignorance of this detail since it was omitted from virtually all discussions of those results in the popular press.) More interesting is her belief that it would obviously be ludicrous to have mathematical laws reinvented by students.

Coincidentally, the very same example was offered by Piaget several decades earlier to argue in favor of this kind of learning. "It is not by knowing the Pythagorean theorem that free exercise of personal reasoning power is assured," he wrote. "It is in having rediscovered its existence and its usage. The goal of intellectual education is not to know how to repeat or retain ready-made truths"; rather, one becomes educated by "learning to master the truth by oneself."[22] When children aren't handed rulers but in effect asked to invent them, when they construct the idea of ratios for themselves, when they recreate the marvelously consistent relation among the three sides of a right triangle (and discover its relevance to real-world design issues), then they are really learning.

By thinking through the possibilities, students come up with their own ways of finding solutions. They have to invent their own procedures. What that means in practice is as straightforward as it is counterintuitive: teachers generally refrain from showing their classes how to do problems. Rather than demonstrating the "correct" procedure for adding two-digit numbers, for example, second-grade teachers might pose a problem and then let the students (individually or in pairs) find ways to solve it, encouraging them to try various techniques, giving them ample time before calling them back together for a discussion so they can explain what they did, challenge each other's answers (in a friendly, supportive way), ask questions, reconsider their own approaches, and figure out what works and why.

This approach has been described in detail by Constance Kamii, a leading student of Piaget's, in a series of three books about how children in first, second, and third grade, respectively, can "reinvent arithmetic." Ultimately, of course, it matters whether students come up with the right answer, but if they're led to think that's all that matters, they're unlikely to understand what's going on. Thus, says Kamii, "if a child says that 8+5=12, a better reaction would be to refrain from correcting him and . . . ask the child, 'How did you get 12?' Children often correct themselves as they try to explain their reasoning to someone else."[23] Because that "someone else" can be a peer, it often makes sense for children to explain their reasoning to one another. Moreover, just as correction of wrong answers isn't especially useful, neither is praise for right answers. Again, what matters is the process – or, more accurately, the child's grasp of the process, which can be elicited by the question "How did you get 13?"[24]

The teacher in such a classroom has a very difficult job. He has to bite his tongue a lot, and also refrain from having children put their answers down on paper too early, since that can get in the way of really thinking through the problems. He has to know when to challenge students: if they all come up with the same method and the right answer, he'd probably be inclined to ask, "Is that the only way you can do it?" (The quality of math instruction at any grade level can almost be measured as a function of how often that question is asked.)

I believe this whole approach makes sense for four reasons. First, it reflects the rock-bottom reality that knowledge about numbers and how they're related can't be taught (that is, given) to children. It has to be constructed. As Kamii comments,

> Educators are under the illusion that they are teaching arithmetic when all they are really teaching are the most superficial aspects such as specific sums (4+4=8, 4+5=9…) and the conventional meaning of written signs (e.g., 4 and +). . . . If a child cannot construct a relationship, then all the explanation in the world will not enable him to understand the teacher's statements. . . . Wrong ideas have to be modified by the child. They cannot be eliminated by the teacher.[25]

Even if a teacher does nothing but demand memorization of facts and practice with procedures, students typically make up and use their own strategies anyway — in effect, constructing their own meaning – sometimes while pretending they're solving problems the way they were told to do so.[26]

Second, explicitly inviting children to make up their own procedures gives the teacher a much better sense of what they understand and what they need help with. An open-ended invitation to tackle a new kind of problem lets the teacher see how they think, whether they can integrate earlier concepts, and exactly where they get stuck – as opposed to judging only whether they got the right answer. Recall that, from a constructivist point of view, one of the most important aspects of a teacher's job is to know as much as possible about each student's thinking.

The third argument for this approach is that it really works. I'll present the scientific research later, but for now, let's listen to a second-grade teacher who announced that she gave away 22 lollipops from a bag of 40. She wanted to know how many were left.

> I watched the children as they struggled to solve the problem mentally. They were extremely quiet. Some of them were staring into space intently, as if they were solving the problem on an invisible chalkboard. Others sat nodding their heads as if in rhythm to the numbers. A few manipulated their fingers, and one child was biting his lip and looking quite puzzled.
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> After several minutes of thinking time had elapsed, most of the children had raised a hand to let me know that they had an answer to share with the group. Their proposed answers were 29, 22, 18, 28, and 12. I wrote each answer on the board without comment, and then I asked, 'Are there any answers here that bother you?'
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> The children immediately got involved in thinking about the answers that had been proposed. . . . Allison quietly said, 'I don't think the answer can be 29, but I'm not sure why.' Many of the children nodded their heads in agreement.
>
> Steve, on the other hand, was not so tentative. He simultaneously raised his hand, stood up, and began to speak. 'Because 29 is too big,' he insisted. If you take 20 from 40, it's 20. So 29 is too high.'
>
> Ben barely waited for Steve to pause and take a breath. He pointed toward the board and authoritatively stated, '40 minus 20 is like 4 minus 2 equals 2. So, 40 minus 20 is 20. Take away 2 more. That's 28.'. . . I was hoping that one of the children would notice the mistake and bring it to Ben's attention. I was not disappointed.
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> Steve commented, 'I don't agree; 20 take away 2 can't be over 20, 'cause you're taking stuff away.'

After more discussion, the children broke up into groups and used blocks (an example of what educators call "manipulatives") to give concrete form to the ideas they'd been discussing. Over the next few weeks, they wrestled with other problems. Ultimately, the teacher reports, "they reinvented regrouping." They not only figured out on their own how to solve such problems but understood the idea behind the method.[27]

Until you see how this works, the idea of trusting children to solve unfamiliar problems – indeed, even the idea that math is a "creative" enterprise involving "invention" – can be very hard to accept. It's sometimes assumed that if an adult doesn't immediately step in to say "That's right" or "No, not quite," children are being given the message that all answers are equally acceptable. In fact, though, not only is it inaccurate to say that a constructivist math classroom is based on that relativistic premise, but exactly the opposite is true. It's precisely the fact that "40 minus 22" has only one right answer that makes this approach work. "Children will eventually get to the truth if they think and debate long enough because, in [math], absolutely nothing is arbitrary," says Kamii.[28] Even the kinds of errors that kids make on the way to understanding reflect certain predictable patterns, not unlike their early spelling mistakes. (For example, young children trying to figure out how many numbers separate 3 from 8 will often start counting at 3 instead of at 4, thereby coming up with an answer that's off by one.)

Along with the charge of relativism, constructivists are sometimes accused of believing that children will just absorb mathematics automatically without the teacher's having to do anything. This, of course, is just another version of the fallacious equation of progressive education with a kind of laissez-faire Romanticism: sit back and children will learn. Because "teaching" is equated with direct instruction in the minds of many traditionalists, the absence of that particular method is taken to mean that the teacher does nothing at all. By now we understand that the teacher is vitally active, integrally involved. She sets things up so students can play with possibilities, think through problems, converse and revise. That's infinitely harder than doing a sample problem and handing out worksheets.

But we can say more than that this approach is effective. The final justification for teaching math this way is that the conventional transmission approach can be positively harmful. A teacher (or parent) for whom the right answer means everything is one who will naturally want to tell the child the most efficient way of getting that right answer. This creates mindlessness. Such a student, armed with algorithms, gets in the habit of looking to the adult, or the book, instead of thinking it through herself. She feels less autonomous, more dependent. Stuck in the middle of a problem, she doesn't try to figure out what makes sense to do next; she tries to remember what she's supposed to do next.[29] That, in a nutshell, is the legacy of traditional education.

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As I've read the literature on nontraditional math education and watched such teaching in action, it has dawned on me that an interesting paradox is at work. On the one hand, it's seen as important to wait until kids are able to understand a concept before introducing it. This caution follows from all of Piaget□s findings about the qualitative changes in children's thinking. You can get toddlers to memorize the words "fifteen squared is two hundred twenty-five." Indeed, I have friends who amuse themselves and their guests by making their children recite impossibly precocious phrases. But these kids might as well be learning nonsense syllables. Parents are often amazed at how early their children can count (in the sense that they can say, "one, two, three, four…") but soon realize that they don't understand the relative quantities signified by each number. In this case, the developmental limitation is clear. But it's not as easy to see – though just as true – in the case of a six-year-old doing double-column addition. He can follow the steps but is almost certainly unable to understand place value – that is, how the tens column is related to the ones column.

A traditionalist like E. D. Hirsch — a college English professor by training, incidentally – feels no compunctions about asserting that "the reported difficulty that American children under age ten have in understanding place value is very likely owing to their lack of consistent instruction and practice in arithmetic."[30] But the truth is that you can force kids to practice until the cows come home and they're still not going to understand what it means to talk about "tens" before they're ready.[31] Thus, there's no point in getting them to do by rote what doesn't make sense to them. All that does is teach them to see math as something people aren't expected to understand.

The paradox is that while constructivists are alert to what children aren't able to do, they also have an unusually generous respect for what children are able to do – spontaneously and at a very young age. "Children's learning begins long before they attend school," noted the famous Russian psychologist Lev Vygotsky. They "have their own preschool arithmetic, which only myopic psychologists could ignore."[32] Myopia isn't really required to miss this, though: all you have to do is make kids memorize facts and follow recipes. Teachers who spend their time telling instead of asking, instructing instead of inviting, can stand in front of children for years without having any idea of what they're capable of doing. A veteran elementary school teacher reflects:

> I had never given children credit for being smart enough to invent solutions. It took a lot of extra effort as a teacher to listen to what they were trying to say, and a lot of self-control to squelch the urge to take the fast-and-easy way of imposing my adult views and methods. But there was so much that never needed to be taught, because the children invented all kinds of things that had not even occurred to me. During math, I now see excitement, enthusiasm, and concentration on the children's faces. I hear voices coming from children who are self-assured, rarely timid, and quiet only while thinking. I find myself wondering how teachers can go on depending on workbooks and drill sheets. But I also recall how skeptical and unsure I was at first of not showing the children how to solve a problem the 'right' way.[33]

To say that teaching from a constructivist perspective is characterized by a paradox – don't give young children more than they can handle, but do give them a chance to show you what they can do – is to put this positively. The flip side is that the Old School manages to screw up on both counts, simultaneously failing to understand children's developmental limitations ("Just drill 'em until they get it") and failing to appreciate their minds ("Use the technique I showed you"). This double fault is what Lilian Katz may have been getting at when she talked about educators who "overestimate children academically and underestimate them intellectually."[34] Nontraditional teachers dedicate themselves to avoiding both traps.

[from Appendix A]

The Hard Evidence: Math Results

Whether U.S. students are doing "well enough" in math is, of course, a judgment call. But to whatever extent we are dissatisfied with their proficiency, there is good reason to believe that the traditional model of instruction bears a large part of the responsibility.

In the mid- to late-1990s, a comprehensive international comparison of math and science teaching, known by the acronym TIMSS, was conducted and then released piecemeal. Part of the study involved a series of conventional tests given to students all over the world in the equivalent of fourth, eighth, and twelfth grades. U.S. students did rather well in fourth grade, rather poorly in eighth grade, and miserably in twelfth grade – although questions have been raised about the data underlying these conclusions.

TIMSS wasn't limited to standardized test results, however; it also included an analysis of curriculum materials and classroom practices. For one segment of the study, James Stigler and his colleagues videotaped more than 200 eighth-grade math teachers to review their methods and also distributed questionnaires to get a sense of the teachers' objectives. Three out of five U.S. teachers said they were chiefly concerned with "skill building." Only one out of four Japanese teachers responded that way: the overwhelming majority said they wanted their students to understand a particular math concept. That goal led those teachers to include deductive reasoning in their instruction, which played a role in 62 percent of Japanese lessons and 0 percent of U.S. lessons. Japanese teachers also explored the intricacies of specific mathematical concepts with their students rather than just naming those concepts, American style. In Japanese classrooms, fewer math problems were considered in more depth, and students participated actively in suggesting different ways of solving those problems. Also, interestingly, homework was rarely assigned.[35]

The overall conclusion reached by the TIMSS researchers – which somehow didn't make it into the headlines, or even into the news stories, when the test results were released – was that traditional forms of teaching, and an emphasis on the basics, contributed significantly to the low standing of older American students. Even before the last phase of the study was reported (looking at the final year of high school, and finding the worst results for U.S. students), the TIMSS authors wrote: "Instruction in this country still seems – compared to instruction in some other countries – more centered on students as passive absorbers of knowledge rather than as active participants who construct, transform, and integrate knowledge."

They went on to predict that "a widespread choice to focus on 'basics'" in American schools would probably lead to "corresponding differences in student achievement, and the differences should be cumulative, with U.S. students falling further behind as they move through the grades" – which is exactly what happened. These findings can't be explained in terms of "natural differences" among students – that is, innate ability – or even in terms of how much effort students put in, since "curricular differences . . . affect how much students can achieve even by working hard." Looking at data from around the world, the researchers found that students lucky enough to live in countries that avoided the "back to basics" approach to instruction "performed comparatively far better" on tests of real understanding.[36]

Recall that these conclusions precisely mirror those of the National Assessment of Educational Progress (NAEP), the major US assessment of student achievement, in terms of math instruction. They're also corroborated by a study of math instruction in the upper elementary grades, which found that "heavy emphasis on skill development and slight attention to concepts and applications may help to explain the United States' relatively poor standing among other nations on mathematics problem-solving ability of students."[37]

Another kind of support came in 1998, when a researcher compared students' use of computers to their NAEP math scores. The overall finding, surprisingly enough, was that the more time students spent on computers during school, the worse they did on that exam. But, on closer inspection, it turned out that nontraditional uses of computers, such as for simulations and learning games, were beneficial. The negative achievement effect was confined to those students who used computers primarily for practicing basic skills.[38] Apparently even the use of new technology is not enough to mitigate the destructive effects of the old pedagogy.

So much for recent evidence on traditional approaches to math instruction. What can we say about efforts to introduce more conceptual and constructivist alternatives? The research conducted on such programs has been concentrated in the primary grades, and it points to a result that can be summarized in six words: better reasoning without sacrificing computational skills – an interesting echo of what we've just seen about a nontraditional approach to teaching reading (namely, better comprehension without sacrificing decoding skills).

In one study, forty first-grade teachers in Wisconsin were given special training in how to make problem solving the organizing focus of teaching arithmetic. When achievement tests taken by their students were later compared to those of traditionally taught children, the results showed a modest, though consistent, edge for the former group. "A focus on problem solving does not necessarily result in a decline in performance in computational skills," the authors wrote.[39]

A few years later, some researchers in Delaware attempted something similar with a smaller group of second-grade teachers. Children in nontraditional classrooms solved a lot fewer problems over the course of the year, but presumably with more thought and understanding because they ended up doing better on tests – particularly when they had to solve problems they hadn't seen before.[40] A third study confirmed that a more conceptual approach to math instruction at both the elementary and secondary levels didn't entail a sacrifice in standardized test scores, even during the transition away from traditional instruction.[41] And a fourth project, in Maryland, found that such math instruction boosted achievement for low-income, mostly minority students – although it took a couple of years for the benefits to reach statistical significance.[42]

Still another group of researchers, at Purdue University in Indiana, developed a very specific framework for teaching the local school district's math objectives to second graders. Problems were presented for children to work on in pairs, after which the whole class came together to talk about what they had come up with. There were no grades, no praise for right answers, no textbooks or worksheets, no requirements for getting through a certain number of problems, and no demonstration by the teacher of the "correct" way to solve them. Teachers took care to encourage productive collaboration and to create a supportive environment where kids could safely challenge one another's ideas.[43]

After a pilot classroom had been set up and analyzed, the researchers were ready to compare the effects of their model to those of traditional classes in three schools. There was virtually no difference in how well the children did basic computation, but those in the alternative classrooms demonstrated significantly higher levels of mathematical reasoning.[44] All the students then spent the next year in conventional classrooms and were tested again. Those whose second grade experience had been nontraditional were still more advanced at conceptually challenging tasks.[45]

Meanwhile, the researchers were extending their project to more second-grade classrooms as well as to some third-grade classrooms, thus allowing them to gauge the effects of spending two consecutive years in a place where "patterns, relationships, and meanings are constituted by students" and where math class is reconfigured as "a community in which mutual exchange and in-depth interaction occurs." The additional year in such a setting did indeed make a difference. Many different kinds of math competence were tested, and while not all measures showed a statistically significant effect, none indicated better performance for traditionally instructed students than for those who had had two years of alternative math. The latter students were far more proficient at understanding problems presented in nontextbook formats, and they also did better at basic computation – even after they had gone on to spend a year in a conventional classroom. Those who had had only one year of constructivist math were more likely to be dragged back down to the level of drill-and-skill students – not only in their achievement but in their attitudes: they came to accept that math is about "solving problems using a single method" rather than believing, as they once had, that it's about "trying to understand and figure out for oneself."[46]

On a smaller and more informal scale, the constructivist theorist Constance Kamii has tested a few elementary classrooms in which children worked all problems on their own, without being given any algorithms. Consistent with the other studies, she discovered that two constructivist second-grade classes did about as well as two conventional classes on a standardized achievement test but performed better on measures of thinking.[47] A subsequent comparison of third graders also found that the “Constructivist Group used a variety of procedures, got more correct answers, and made more reasonable errors when they got incorrect answers. The Comparison Group by and large had only one way of approaching each problem – the conventional algorithm – and tended to get incorrect answers that revealed poor number sense.”[48]

One last point, which is not so incidental: a teacher working with Kamii commented that after she adopted the nontraditional approach to instruction, her classes “displayed a love of math that I had not seen during my first decade of teaching.”[49] While there are no hard data to confirm this impression (as there are with Whole Language), it certainly matches what the Purdue researchers witnessed in their experimental classrooms. They reported that visitors “invariably remarked about the excitement for mathematics displayed by the children as they solved the activities. Children frequently jumped up and down, hugged each other, and rushed off to tell the teacher when they solved a particularly challenging problem.” Moreover, they persisted at difficult problems to an unusual degree and took pleasure in one another’s successes.[50] Of course, this is probably related to the absence of stickers, grades, praise, and other reinforcers that tend to interfere with children’s delight in the learning itself. But the tasks must be sufficiently engaging and open-ended so that success is potentially delightful – something far less likely to happen when children are just expected to go through the approved steps to get the correct answers on a worksheet.

NOTES

[For full citations, please see the Reference section of The Schools Our Children Deserve.]

Brownell, 1928, p. 197. Ibid., p. 200. Lampert, 1986, p. 340. A year-long intensive study of the teaching and learning that took place in a 10th-grade geometry class . . . in a highly regarded suburban school district in upstate New York” produced just such results. While “a classroom observer unfamiliar with mathematics would necessarily give the class high marks,” students basically spent the year copying proofs and then doing exercises “designed to indicate mastery of relatively small chunks of subject matter.” (“Over the period of a full school year, none of the students in any of the dozen classes we observed worked mathematical tasks that could seriously be called problems.”) Indeed, this exemplary teacher, mindful of the standardized test the students would eventually have to take, commented at one point, “You’ll have to know all your constructions cold so you don’t spend a lot of time thinking about them” (Schoenfeld, 1988, pp. 145-46, 152, 159). The National Research Council is quoted in Battista, 1999, p. 427. Resnick, 1987, p. 14. Brownell, 1932, p. 10. He added: if arithmetic somehow does become “meaningful, it becomes so in spite of drill” (p. 12). Putnam et al., 1990, p. 85. This real-life example comes from Paul Cobb by way of Gardner, 1991, p. 164. This example, from Max Wertheimer’s Productive Thinking, is cited in Schoenfeld, p. 148. Kamii, 1994, pp. 36-40. Cited in Schoenfeld, p. 150, among other places. Another example: a startling number of young children taught in traditional classrooms give the answer “36” to the question “There are 26 sheep and 10 goats on a ship. How old is the captain?” (Kamii, 1989, p. 160). Dossey et al., 1988, pp. 67, 54. Because really bright students generally learn symbolic algorithms quickly, they appear to be doing fine when their performance is measured by standard mathematics tests. But a closer look reveals that they too are being dramatically affected by the mathematics miseducation of traditional curricula . . . [ending up with learning that is] only superficial” (Battista, p. 426). Kamii (e.g., 1994, pp. 43, 46) is especially persuasive on this point. Putnam et al., 1990, p. 96. Jackson, 1997, p. I: 1. Kamii, 1985b, p. 3. For examples of fortuitous events that can provide the opportunity for children in first grade, second grade, and third grade to think about numerical concepts, see Kamii, 1985b, pp. 123-35; 1989, pp. 91-97; and 1994, pp. 92-98. Like some other constructivists, Kamii also swears by the use of certain games — such as those involving dice or play money — for teaching purposes. All the games in question, however, are competitive, suggesting both a lack of familiarity with cooperative games (where the same numerical skills are often required) and a lack of sensitivity to the social and psychological disadvantages of setting children against each other (see Kohn, 1992a). Joseph Kahne at the University of Illinois, Chicago, makes this point. As a rule, he argues, parents “aren’t nervous about Whole Language because they know their kids will be reading; their literacy skills aren’t threatened” (personal communication, 1997). However, this may understate the extent to which Whole Language diverges from most people’s school experience. One writer notes: “Sadly, many parents don’t recall being given the opportunity to read ‘real books’ in their early elementary classrooms unless (as in my case) it was after all their ‘work’ was done. Thus, attacks on whole language that focus on literature grow partly from parents’ discomfort that their children’s school experience isn’t like their own” (Brinkley, 1998, p. 59). And this from another writer: “When you take away the two school rituals that parents understand – math facts and spelling quizzes – you scare them to death” (Ohanian, 1996a, p. 9). The washer lesson is described in Brooks and Brooks, 1993, pp. 73-75. The bubblegum lesson was used by Pam Hyde and appears in Zemelman et al., 1998, pp. 85. The fraction problem comes from Joy Donlin and is reported in Willis and Checkley, 1996, p. 7. The idea of having students write a textbook is attributed to Bill Elasky by Wood, 1992, p. 140. Kahramanidis, 1988. Piaget, 1973, p. 106. Kamii, 1985b, p. 46. Another, more practical reason for asking this same question about a correct answer is that otherwise children will just assume “How did you get that?” is teacher code for “Nope — try again.” Kamii, 1985b, pp. 25, 36. Her constructivist premises have led Kamii to offer only a partial endorsement for the NCTM standards. She argues that, despite their emphasis on deeper understanding of mathematical truths, the standards still reflect an empirical view that those truths have a reality entirely independent of the knower. Further, while collaboration among students is recommended, Kamii believes the standards fail to reflect a constructivist appreciation for the necessity of understanding through resolving conflict among disparate ideas (see Kamii, 1989, pp. 59-62). This point was made by Brownell, 1928, pp. 199, 208-9; and also by Jean Lave, cited in Brown et al., 1989, p. 36. Unfortunately, students in this situation aren’t being appropriately challenged (by the teacher or other students) to rethink and improve their initial ideas, so they probably won’t learn as effectively as they would in a nontraditional classroom. Lester, 1996, pp. 146-52. For another teacher’s description of how – and how well – this approach works, see Strachota, 1996, chap. 3. Kamii, 1994, p. 67. Not every math educator agrees that primary-grade children shouldn’t be given algorithms at all, but Kamii makes a strong case for this position. Rob Madell (1985, p. 20) similarly recommends that no algorithms be taught until the end of third grade, and no conventional procedures for working with fractions be introduced until sixth grade (even though students will have studied fractions intensively for at least two years before that). Hirsch, 1996, p. 83. “Research has shown, however, that most children think that the 1 in 16 means one, until third or fourth grade” (Kamii, 1989, p. 15). “Even in fourth and fifth grades, only half the students interviewed demonstrated good understanding of the individual digits in two-digit numerals” (Ross, 1989, p. 50). Vygotsky, 1978, p. 84. Linda Joseph’s account appears in Kamii, 1989, p. 156. A nearly identical piece of testimony from another teacher – “I have been teaching all this time [fifteen years] and I never knew second-graders knew so much about math” – is quoted in another discussion of what it means to become a constructivist math teacher (Wood et al., 1991, p. 601). Katz, 1993, p. 31. Also see Katz and Chard, 1989, pp. 4-5. See Stigler and Hiebert, 1997; and Lawton, 1997. Other differences between U.S. and Japanese instruction may also contribute to differences in results. Japanese teachers meet regularly in small groups so they can collaboratively evaluate their teaching and improve their craft (Stigler and Hiebert, p. 20). Also, Japan, like many other countries, does not “track” students by putative ability (Schmidt et al., 1998). Schmidt et al., 1998, pp. 10, 15, 18, 25-6. Porter, 1989, p. 11. Mathews, 1998b. The report, prepared by Harold Wenglinsky of the Educational Testing Service, also found that African-American children were especially likely to use computers for drill-and-skill purposes. Once again, a back-to-basics approach to instruction is disproportionately used for children of color – to their detriment. Carpenter et al., 1989. Quotations appear on pp. 525, 527. Hiebert and Wearne, 1993. Simon and Schifter, 1993. Campbell, 1996. Yackel et al., 1991. Cobb et al., 1991. Cobb et al., 1992. Wood and Sellers, 1996,

1997. Kamii, 1989, pp. 158-78. Kamii, 1994, p. 205. Linda Joseph, quoted in Kamii, 1989, p. 155. Cobb et al., 1989, pp. 137, 139, 144.